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Impact of detuning and dephasing on a laser-corrected subnatural-linewidth single-photon source

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Abstract. The elastic scattering peak of a resonantly driven two-level system has been argued to provide narrow-linewidth antibunched photons. Although independent measurements of spectral width on the one hand and antibunching on the other hand do seem to show that this is the case, a joint measurement reveals that only one or the other of these attributes can be realised in the direct emission. We discuss a scheme which interferes the emission with a laser to produce simultaneously single photons of subnatural linewidth. In particular, we consider the effect of dephasing and of the detuning between the driving laser and/or the detector with the emitter. We find that also in presence of dephasing, our scheme brings considerable improvement as compared to the standard scheme.

1. Introduction

Of the many schemes to implement a single-photon source [1–20], various strategies are adopted to favour one or the other of its desirable characteristics, ranging from its size and ability to interface with other optical devices [1–4], its brightness [5–11], the indistinguishability between successive photons [11–17] and, of course, its sub-Poissonian character [18–20]. More exotic schemes have also been proposed, such as the photon blockade, in both its conventional [21, 22] and unconventional [23–25] versions (both of which have been recently demonstrated experimentally [26–29]) or the heralding of single photons from two-level systems driven in Mollow triplet regime [30], let alone sources of N -photons “bundles” [31, 32]. One of the most popular platforms for the generation of single photons, both from an experimental and theoretical point of view, is a two-level system. This can be realised in a variety of platforms ranging from cold atoms [33–35], to semiconductor quantum dots [12, 36–43] passing by ions [44–46], molecules [47–50], superconducting circuits [51–55], nitrogen vacancies [56–58], among others. A priori, a two-level system fits perfectly the purpose, as it can only sustain a single excitation at any given time. Thus, its repetition rate is limited by the time it takes to “reload”, and one can expect a perfectly antibunched emission. This is however a simplified description that ignores a central aspect of

quantum theory: the detection process. The two-level system is characterized to the best of its abilities only by a detector that can measure its emission with infinite precision in time. Conversely, if the detector has a finite temporal resolution (as is of course the case in any actual setup), or, equivalently, a finite bandwidth, the theoretically perfect suppression of the second-order correlation function is spoiled by the Heisenberg uncertainty principle [59]. This can be described accurately by the theory of frequency-filtered correlations [60].

While the impact of detection is a fundamental principle that applies to all quantum systems, an interesting and somehow counter-intuitive effect occurs when turning to the detected emission of a two-level system driven coherently in the so-called Heitler regime [61], in which the emission of a two-level system consists of two components: i) photons that are absorbed and later re-emitted (fluorescence) and ii) photons that are elastically scattered by the two-level system (in a coherent absorption and re-emission process). The former are emitted with a Lorentzian profile centered at the frequency of the driving laser and with the natural linewidth of the two-level system, constituting the incoherent fraction of the emission. The latter are emitted as a δ -narrow peak (assuming a vanishing linewidth for the laser), which forms the coherent fraction of the emission that dominates at low driving. Like any two-level system, the total emission is antibunched. The idea then arose to use the δ peak to collect antibunched photons with narrow spectral width [62, 63]. Here as well, one must not forget the process of detection, and taking it into account, we have shown that these two qualities are not realised jointly [64]: the detected photons are either antibunched, but with a spectral width no better than that of the emitter itself, or they can be detected with the spectral bandwidth of the δ peak, but then their antibunching is dramatically reduced. Interestingly, however, we have shown in the same work [64] how to detect photons jointly with a subnatural linewidth and an excellent antibunching, by interfering the emission of the filtered two-level system with an external laser. This laser correction removes, through destructive interferences, the excess of coherent emission when focusing on the δ peak, in a process akin to an homodyne interference [65, 66]. Similar schemes have been recently implemented to obtain a source of indistinguishable photons [14], to observe the rising of the so-called dynamical Mollow triplet [67] and to unveil the photon correlations of the light emitted by a Jaynes-Cummings system [68]. In our case, we find that not only this laser-correction allows to realise simultaneously subnatural linewidth spectral emission and antibunching, but also that it produces a stronger type of single-photon emission with a plateau in the time-resolved photon correlation $g_a^{(2)}(\tau)$. Such sources therefore provide a new playground of their own, whose properties, advantages over existing sources and further possibilities deserve an immediate attention, as we wait for their experimental implementation.

In this text, we provide a more general picture, including other interesting features of the statistics, such as perfect superbunching (where, to first order in the driving, $g_a^{(2)}(0)$ becomes infinite) in addition to the previously reported perfect antibunching ($g_a^{(2)}(0) = 0$). More particularly, we focus on the effect of two important aspects not considered previously: the impact of dephasing, since this is a detrimental ingredient that is typically present, especially in a solid-state platform, and the role of detuning from the two-level system, from either the driving laser and/or the detector.

2. Theoretical description

We consider a two-level system driven by a coherent source in the regime of low excitation, commonly referred to as the Heitler regime, with Hamiltonian (we take $\hbar = 1$ along the text)

$$H_\sigma = (\omega_\sigma - \omega_L)\sigma^\dagger\sigma + \Omega_\sigma(\sigma^\dagger + \sigma). \quad (1)$$

The two-level system has a frequency ω_σ and is described through an annihilation operator σ that follows the pseudo-spin algebra, whereas the laser is treated classically, i.e., as a c -number, with intensity Ω_σ and frequency ω_L . The dissipative character of the system is included in the dynamics through a master equation

$$\partial_t \rho = i[\rho, H_\sigma] + \frac{\gamma_\sigma}{2}\mathcal{L}_\sigma\rho + \frac{\gamma_\phi}{2}\mathcal{L}_{\sigma^\dagger\sigma}\rho, \quad (2)$$

where γ_σ is the decay rate of the two-level system, γ_ϕ is the dephasing rate, H_σ is the Hamiltonian in Eq. (1) and $\mathcal{L}_\sigma\rho = 2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma$. The steady-state solution of Eq. (2) can be written in terms of two magnitudes: the population, $\langle\sigma^\dagger\sigma\rangle \equiv n_\sigma$, and the coherence, $\langle\sigma\rangle \equiv \alpha$, of the two level system:

$$\rho_{\text{ss}} = \begin{pmatrix} 1 - n_\sigma & \alpha \\ \alpha^* & n_\sigma \end{pmatrix}, \quad (3)$$

where

$$n_\sigma = \frac{4(\gamma_\sigma + \gamma_\phi)\Omega_\sigma^2}{\gamma_\sigma[(\gamma_\sigma + \gamma_\phi)^2 + 4\Delta_\sigma^2] + 8(\gamma_\sigma + \gamma_\phi)\Omega_\sigma^2}, \quad (4a)$$

$$\alpha = \frac{2i\gamma_\sigma\Omega_\sigma[2i\Delta_\sigma - (\gamma_\sigma + \gamma_\phi)]}{\gamma_\sigma[(\gamma_\sigma + \gamma_\phi)^2 + 4\Delta_\sigma^2] + 8(\gamma_\sigma + \gamma_\phi)\Omega_\sigma^2}, \quad (4b)$$

with $\Delta_\sigma = (\omega_\sigma - \omega_L)$ the detuning between the two-level system and the driving laser.

To model the detection process self-consistently, one can couple to this system a detector and study the observables through this detector rather than from the object itself. A method (the so-called ‘‘cascaded formalism’’) has been developed in the early 90s [69, 70] to model this theoretically. Such a precaution avoids (or reveals) the subtle mistake of assuming that the emitted light retains all the attributes of the source when these are computed or measured separately. To take the example of interest in our discussion, one can indeed observe (or compute) the spectral width of the two-level system in the Heitler regime, and find an ultra-bright and ultra-narrow component, and then observe (or compute) its antibunching and find an excellent antibunching. These constitute separate characterisations of the source, and until these are measured simultaneously, they cannot be assumed to exist simultaneously. Indeed, characterising the light through the detector—which by the very nature of its excitation is being subject to the both aspects of interest simultaneously—one finds that the detected light is either antibunched but not narrower than the source, or is spectrally narrow but then with a poor or no antibunching. This depends on the spectral width of the detector itself: if the detector has a large spectral width, it will not be sensitive to the supposedly narrow linewidth light that excites it. If the detector has a narrow spectral width, it will not be sensitive to its antibunching. Given the character of quantum mechanics, we conclude that the failure of a detector to simultaneously capture the narrow linewidth and the antibunching really means that these do not actually jointly exist.

In the following we will discuss and generalise a scheme which we have recently proposed [64] and that achieves such a joint narrow and antibunched emission, in the

sense that a detector does collect its light with these two attributes intact. As we will focus on antibunching and spectral width, we can support our analysis of the detection process through a “sensor” that acts as a filter for the emitted light [60]. Theoretically, this is included through the vanishing coupling of a bosonic field with annihilation operator a to the dynamics of the two-level system, by adding the Hamiltonian $H_a = g(\sigma^\dagger a + a^\dagger \sigma)$ to Eq. (1) and then taking the limit $g \rightarrow 0$, which allows the dynamics of the two-level system to be independent from that of the sensor. The bandwidth of the sensor is given by its decay rate Γ and is included as an extra term $(\Gamma/2)\mathcal{L}_a\rho$ in the master equation (2). For quantities such as populations, which would vanish with $g \rightarrow 0$, one should use instead the more complete but also heavier cascaded formalism, which we have shown is equivalent to the more lightweight sensor method as far as correlations are concerned [71]. The main point of this theoretical shortcut is that instead of considering the light emitted by a system, one can consider instead the filtered light and this is enough to describe the process of detection, as long as the detector would have the same spectral width as the filter. We will therefore be speaking of filtering for the light emitted by the two-level system, which should be understood as the effect of its detection from a detector with the corresponding bandwidth.

When the emission of the two-level system is filtered in frequency, the tails of the incoherent Lorentzian are trimmed out and this spoils the perfect antibunching, that arises from an interference between the coherent and incoherent components [64]. Such an interference can be restored simply by reinstating the original proportion, i.e., the perfect antibunching can be maintained after filtering by reducing the surplus of coherent emission that passes in its entirety through the filter. This can be easily achieved since a coherent field can be scaled at will through interferences, in our case, destructive interferences. Our scheme thus consists of interfering at a beam splitter the light emitted by the two-level system with a coherent field $\beta \equiv |\beta|e^{i\phi}$, whose amplitude and phase need to be fixed adequately to provide the exact compensation. In this configuration, the Hamiltonian for our laser-corrected source becomes

$$H = H_\sigma + \Delta_a a^\dagger a - ir|\beta|(e^{i\phi} a^\dagger - e^{-i\phi} a) + gt(\sigma^\dagger a + a^\dagger \sigma), \quad (5)$$

where H_σ is the Hamiltonian in Eq. (1), t and r are the transmission and reflection coefficients of the beam splitter (the reflection coefficient is preceded by a factor i , which accounts for the phase shift gained by the reflection in the beam splitter), and $\Delta_a = (\omega_a - \omega_L)$ is the detuning between the detector and the driving laser. The amplitude of the coherent field $|\beta|$ can be parameterised as a fraction \mathcal{F} of the coherent field that the sensor receives from the two-level system, i.e., we may write

$$|\beta| = g \frac{\Omega_\sigma}{\gamma_\sigma} \frac{t}{r} \mathcal{F}, \quad \text{or equivalently} \quad |\beta'| = g \frac{\Omega_\sigma}{\gamma_\sigma} \mathcal{F}, \quad (6)$$

where we have also defined $|\beta'| = |\beta|(r/t)$, to avoid carrying the parameters of the beam splitter, which only renormalize the observables but do not change the physics involved.

3. Results

The two-photon correlations detected in various spectral widths are obtained from the steady-state solution of the master equation (2) with H_σ replaced by the Hamiltonian in Eq. (5) and with the added Lindblad term $(\Gamma/2)\mathcal{L}_a\rho$. Although these correlations

can be obtained in closed-form, they are too cumbersome to be written here. Instead, we will provide the particular cases i) with detuning but no dephasing and ii) at resonance but with dephasing. The full case, of which we will show one case graphically, brings little more insights, so treating these separately is enough to reach general conclusions. We consider first the case without dephasing.

3.1. No dephasing

In the case of no dephasing, but allowing for some detuning, either between the driving laser and the two-level system or between the detector and the two-level system (or both), the detected two-photon correlations are given by, to leading order in the driving strength:

$$g_a^{(2)} = \tilde{\Gamma}_\sigma^2 (16\gamma_\sigma^4 + 16\mathcal{F}^2\gamma_\sigma^2\tilde{\Gamma}_+^2 + \mathcal{F}^4\tilde{\Gamma}_\sigma^2\tilde{\Gamma}_+^2 - 8\mathcal{F}\gamma_\sigma\{2(4\gamma_\sigma^2\Delta_+ + \mathcal{F}^2\Delta_\sigma\tilde{\Gamma}_+)\sin\phi + 2\gamma_\sigma\mathcal{F}(\gamma_+\Delta_\sigma + \gamma_\sigma\Delta_+)\sin 2\phi - \gamma_\sigma(4\gamma_\sigma\gamma_+ + \mathcal{F}^2\tilde{\Gamma}_+^2)\cos\phi - \gamma_\sigma\mathcal{F}(\gamma_\sigma\gamma_+ - 4\Delta_\sigma\Delta_+)\cos 2\phi\}) / (\tilde{\Gamma}_+^2[4\gamma_\sigma + \mathcal{F}^2\tilde{\Gamma}_\sigma + 4\gamma_\sigma\mathcal{F}(\gamma_\sigma\cos\phi - 2\Delta_\sigma^2\sin\phi)]^2), \quad (7)$$

where we have defined $\gamma_+ \equiv \gamma_\sigma + \Gamma$, $\Delta_+ \equiv \Delta_\sigma + \Delta_a$ and $\tilde{\Gamma}_c^2 \equiv \gamma_c^2 + 4\Delta_c^2$ for $c = \sigma, +$. From this expression, it is easy to find particular cases of interest: perfect antibunching, when the numerator becomes zero, and perfect superbunching, when the denominator becomes zero.

The condition to vanish the numerator, and therefore to produce perfect antibunching, is given by

$$\mathcal{F} = -\frac{2\gamma_\sigma e^{-i\phi}}{\gamma_\sigma + 2i\Delta_\sigma} \left(1 \pm \sqrt{\frac{\Gamma + 2i\Delta_a}{\Gamma_+ + 2i\Delta_+}} \right), \quad (8)$$

which generalizes the expression given in Eq. (10) of Ref. [64]. Since a real phase ϕ can always be found so that Eq. (8) is a real positive number, the condition for perfect antibunching is always satisfied. The condition that cancels Eq. (7) also yields the suppression of the state with two photons in the detector, as we show in [Appendix A](#) through a wave-function approximation. In the most natural configuration where the laser is resonant to both the cavity and the two-level system, Eq. (8) reduces to the expressions of Ref. [64]:

$$\mathcal{F}_{2,\pm} = 2 \left(1 \pm \sqrt{\frac{\Gamma}{\Gamma + \gamma_\sigma}} \right) \quad \text{and} \quad \phi_+ = \pi. \quad (9)$$

On the other hand, the denominator of Eq. (7) vanishes when one sets the phase ϕ and amplitude \mathcal{F} of the interfering laser to satisfy

$$\tan\phi = -\frac{2\Delta_\sigma}{\gamma_\sigma} \quad \text{and} \quad \mathcal{F} = -2\cos\phi. \quad (10)$$

Note that unlike for antibunching, this condition to obtain superbunching is independent of the detuning between the detected light and the driving laser. In this case, $g_a^{(2)}$ diverges (to first order in the driving strength, so that higher order terms would produce huge but finite values of $g_a^{(2)}(0)$). The appearance of such a strong superbunching from a two-level system admixed with a coherent field is interesting. Its nature is discussed elsewhere [72]. Here, we will content to say that this arises from an interference, as a result of which the photon emission is not occurring in the form

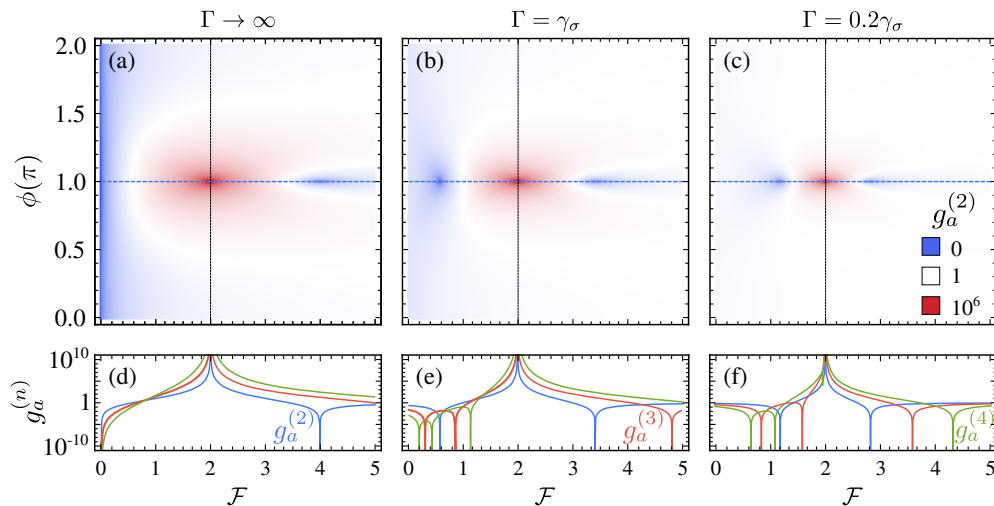


Figure 1. (Color online) Filtered two-photon correlations at resonance and without dephasing, as a function of the parameters \mathcal{F} (intensity) and ϕ (phase) of a superimposed interfering laser. In all the panels we set γ_σ as the unit, $\Delta_\sigma = \Delta_a = 0$ and $\gamma_\phi = 0$. The detector linewidth decreases in from (a) to (c) as indicated on each panel.

of photon bundles and cannot be Purcell-enhanced, so that prospects for applications as N photon sources are limited. Also, we mention that infinite bunching has already been reported before; it occurs for instance with Fock states in bosonic cascades [73].

A full map of $g_a^{(2)}(0)$ as defined by Eq. (7) is shown in Figure 1(a-c) for spectral widths of the detector ranging from essentially full-bandwidth, panel (a), the linewidth of the two-level system, panel (b) and with sub-linewidth resolution, panel (c). In the first case, without frequency filtering, perfect antibunching is obtained without any laser correction, that is, for $\mathcal{F} = 0$ and independently of the phase. This corresponds to the case considered in the literature [62, 63], but this comes at the cost of the spectral width: the δ -width of the laser is completely washed out by the detector. For a detector spectrally matched to the emitter, shown in (b), antibunching is considerably reduced by the detector (to $g_a^{(2)}(0) \approx 0.25$). Keeping the same linewidth, antibunching can be restored by the interfering laser fulfilling conditions (9), restoring an exact antibunching, $g_a^{(2)}(0) = 0$, to first order in the driving. Going to sub-natural linewidth with a detector spectrally matched to $0.2\gamma_\sigma$, shown in panel (c), one finds that antibunching is now almost completely gone in absence of the laser correction, $g_a^{(2)}(0) \approx 0.7$, but can again be fully restored with the laser correction. In all cases, in between the two conditions for antibunching, one can see the superbunching at $\mathcal{F} = 2$. Note that, as the linewidth is made narrower, the conditions for antibunching, Eq. (9), come closer to the conditions for superbunching, Eq. (10). So while this effect could be pursued down to extremely narrow linewidths, at no cost for the antibunching, some restrictions would arise from the stability of the driving laser, as energy fluctuations would take the system from the condition for perfect antibunching to the condition for superbunching. Panels (d-f) show in blue lines transverse cuts at $\phi = \pi$ of panels (a-c), and in red and green lines the corresponding higher-order correlators $g_a^{(3)}$ and $g_a^{(4)}$, respectively. This shows how, although both $\mathcal{F}_{2,\pm}$ from Eq. (9) yield an

exact cancellation of $g_a^{(2)}$, the higher order correlations remain sub-Poissonian in the vicinity of only $\mathcal{F}_{2,-}$ (which corresponds to the condition to obtain a ‘‘conventional antibunching’’ [72]).

3.2. Dephasing

We now turn to the impact of dephasing, which is detrimental to photon correlations, but can still be corrected to a considerable extent through our process, although not perfectly anymore. In the case of dephasing alone, where both the two-level system and the detector are resonant with the driving laser, the two-photon correlations are given by

$$\begin{aligned}
g_a^{(2)} = & \Gamma_\phi(\Gamma + \Gamma_\phi) \left(16\gamma_\sigma^3(\Gamma + \gamma_+)(2\Gamma + \gamma_+) + 16\mathcal{F}^2\gamma_\sigma\gamma_+^2(2\Gamma + \Gamma_\phi)(3\Gamma + \Gamma_\phi) + \right. \\
& + \mathcal{F}^4\gamma_+\Gamma_\phi(\Gamma + \Gamma_\phi)(2\Gamma + \Gamma_\phi)(3\Gamma + \Gamma_\phi) + \\
& + 8\mathcal{F}\gamma_\sigma\gamma_+(3\Gamma + \Gamma_\phi) \{ \mathcal{F}\gamma_\sigma(2\Gamma + \Gamma_\phi) \cos 2\phi + \\
& + [4\gamma_\sigma(\Gamma + \gamma_+) + \mathcal{F}^2(\Gamma + \Gamma_\phi)(2\Gamma + \Gamma_\phi)] \cos \phi \} / \\
& \left. \{ \gamma_+^2(2\Gamma + \Gamma_\phi)(3\Gamma + \Gamma_\phi) \times \right. \\
& \left. [(4 + \mathcal{F}^2)\gamma_\sigma\gamma_+ + \mathcal{F}^2\gamma_\phi(\gamma_+ + \Gamma_\phi) + 4\mathcal{F}\gamma_\sigma(\Gamma + \Gamma_\phi) \cos \phi]^2 \right\}, \tag{11}
\end{aligned}$$

where we have used the notation $\Gamma_\phi = \gamma_\sigma + \gamma_\phi$ and $\gamma_+ = \gamma_\sigma + \Gamma$.

The general case that also includes detuning is shown in Fig. 2. In this case, the laser is detuned from the two-level system, while the detector is set either at the frequency of the two-level system (panels (a) and (c)) or at the frequency of the laser (panels (b) and (d)). The top row of Fig. (2) is given by Eq. (7) while the bottom row, for which a closed-form expression exists but is too bulky to be written here, is only shown graphically. The filter linewidth, or, equivalently, the bandwidth of the detector, has been taken to match one fifth of the emitter linewidth. Therefore, Fig. 2(a) and Fig. 2(b) are the detuned versions of Fig. 1(c). Using this panel as a reference, one can see the impact of dephasing (spoiling the correlations) and detuning (maintained but for different laser corrections). Actually, the condition for superbunching is independent from the detuning between the detector and the driving laser, in agreement with Eq. (10), unlike the condition for antibunching. In fact, when the detector is resonant to the laser, both being detuned from the emitter, the conditions for perfect antibunching do not occur at the same phase ϕ . Therefore, in a detuned measurement, the correlations are easier to observe when the cavity is resonant to the two-level system.

While in absence of dephasing (Eq. (7)), correlations can range from exactly zero to infinity, Eq. (11) shows that in its presence, they can only be pushed to finite values, both for bunching and antibunching. Although the full expressions to obtain such limiting case can be found, they are too bulky to be written here but they are plotted in Fig. 2(e). Similarly, the loss of antibunching due to dephasing cannot be compensated exactly by the interfering laser and for strong enough dephasing, no antibunching at all can be maintained. Nevertheless, the laser correction still brings considerable improvement on the case without interference, which is also, of course, affected by dephasing. The intensity \mathcal{F} that yields the best antibunching is found by minimising Eq. (11), which does not provide a simple closed-form expression, but can be readily found numerically. The minimum antibunching obtained this way is shown in Fig. 2(f).

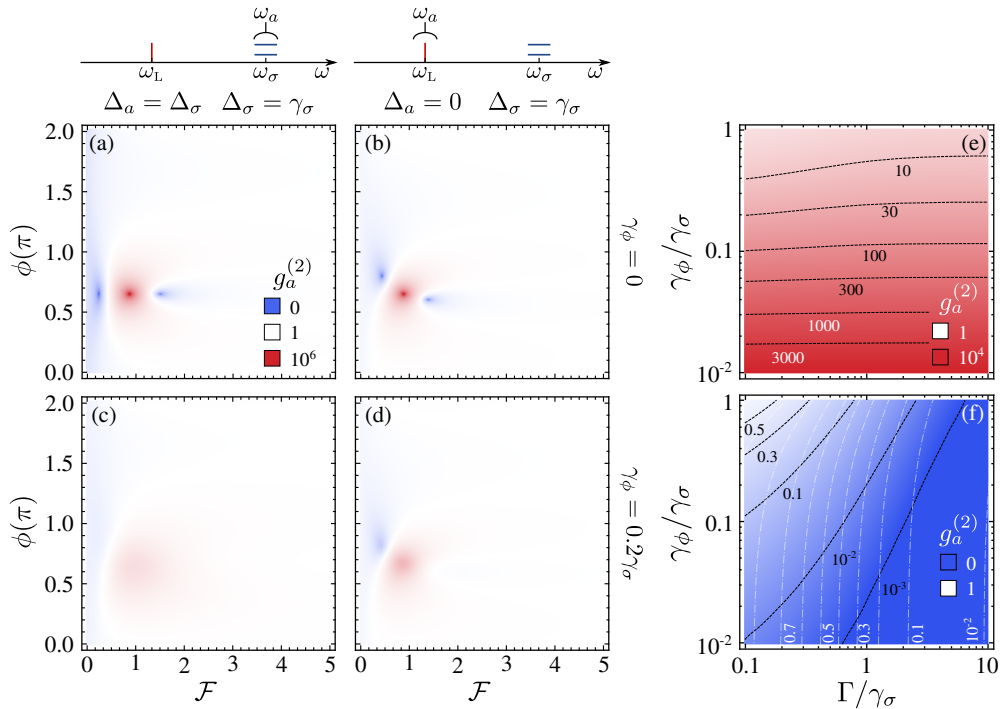


Figure 2. (Color online) (a–d) Two-photon correlations as a function of the parameters of the interfering laser when the laser is detuned from the two-level system, in absence (top row) and in presence (bottom row) of dephasing. In the left column, the detector is resonant to the emitter while in the right column, it is to the driving laser. Dephasing spoils the perfect antibunching, and it is particularly detrimental in the case where the cavity is set in resonance the two-level system, in which case the dephased correlations are completely blurred. Without dephasing, one may still find the condition for \mathcal{F} that provides perfect antibunching. (e,f) Maximum and minimum value of $g_a^{(2)}$ when the two-level system is dephased with $\gamma_\phi = 0.2\gamma_\sigma$. In panel (f) we also show in dashed-dotted white lines the isolines for the minimum correlations that can be obtained without the correction (i.e., with $\mathcal{F} = 0$). Throughout, $\Gamma = \gamma_\sigma/5$.

3.3. Correlations in time

While the value of the zero-delay correlation between photons is usually the one considered to quantify the sub-Poissonian character of a source, the correlations between photons detected with a time difference τ are also important. In particular, fast oscillations in correlations can be difficult to resolve and average out the result. In Ref. [64], we showed that in the case without dephasing and in resonance, when the loss of antibunching due to filtering is corrected with an external laser, the $g_a^{(2)}(\tau)$ displays a plateau of perfectly antibunched photons for up to $|\tau| \approx 2.5\gamma_\sigma$. This actually confers to such sources an even greater single-photon source character. As can be expected, correlations in time are affected by dephasing as well as by the detuning between the two-level system, the cavity and the laser. Such a characteristic profile is shown as filled blue lines in Fig. 3, to which we compare the cases treated in this text. Panels (a) and (b) show the effect of detuning (without dephasing) and Panel (c) shows the effect of dephasing (without detuning). In (a) the detector is resonant with the two-level

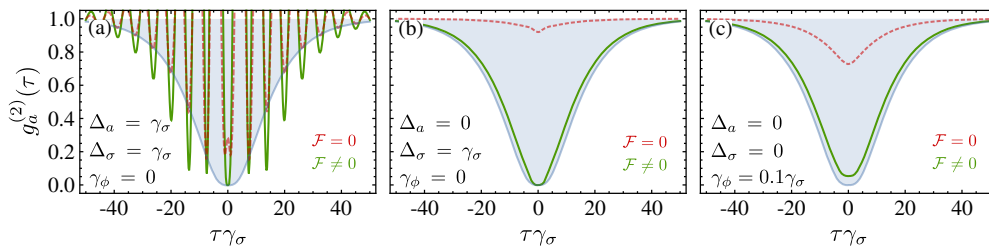


Figure 3. (Color online) Time-resolved filtered two-photon correlations in the various configurations discussed in the text. The filled-blue line is the case at resonance and without dephasing. Panels (a) and (b) show the impact of various detunings without dephasing and Panel (c) shows the impact of dephasing at resonance. In green the best correction that can be achieved with an interfering laser, in red the uncorrected case. In all cases $\Gamma = \gamma_\sigma/5$.

system, and both are detuned from the driving laser, while in (b), the detector is resonant with the laser, and both are detuned from the emitter. In green lines are shown the correlations for the given parameters featured in inset while red lines show the result without the laser correction, i.e., with $\mathcal{F} = 0$.

In Fig. 3(a), when the driving laser is detuned, the zero delay correlations can be compensated exactly, but the antibunching plateau is lost or at least strongly reduced, and the correlations display fast oscillations between antibunching and bunching. Conversely, when the detector is resonant to the laser, panel (b), the plateau of antibunching is still present, albeit for a shorter time, and the correlations do not display any oscillations. This means that, although the configuration with the detector in resonance to the emitter could seem the more natural or appealing one, the fast oscillations in its delay correlations makes it potentially problematic, and if it would turn out to be impossible to bring the driving laser in resonance with the emitter, then the detection should be done at the frequency of the laser. It is also shown how the laser-correction makes a huge improvement on the antibunching as compared to the standard case which features almost no antibunching. However, a very large detuning between the two-level system and the laser washes out the plateau and the correlations become simply $g_a^{(2)}(\tau) = (1 - e^{-\Gamma\tau/2})^2$, which corresponds to the correlations of a two-level system of linewidth Γ driven coherently in the regime of low excitation. In Fig. 3(c), it is shown how the zero-delay correlation of the dephased two-level system *cannot* be compensated exactly in presence of dephasing, as already stated, but otherwise suffers little in term of its plateau or coherence time. More importantly, it remains largely improved as compared to the case without the interference, with a value of $g_a^{(2)}(0) \approx 0.05$ for a dephasing rate of 10% the emitter decay rate, whereas it is only $g_a^{(2)}(0) \approx 0.73$ without the laser correction. Furthermore, the measured linewidth remains well below the natural (but broadened by the dephasing) linewidth of the two-level system.

4. Conclusions

We have discussed our scheme to produce joint antibunching and subnatural linewidth emission [64] in the presence of dephasing and of a detuning from the emitter with either the driving laser and/or the detector. Not only does this scheme makes these

properties hold simultaneously, it also produces perfect antibunching to first order in the driving (that is, $g_a^{(2)}(0) = 0$) which is otherwise possible only by integrating all frequencies, and features a plateau in the time-delayed photon correlations, making such a single-photon source more effectively suppressing coincidences. We have also shown that different conditions from the interfering laser produce instead perfect superbunching (that is, $g_a^{(2)}(0) = \infty$ to first order in the driving).

We have shown how, in presence of detuning alone, perfect antibunching can always be enforced by the interference, but result in strong time oscillations of the statistics when the laser is detuned and the detector is resonant to the laser. On the other hand, no such oscillations are featured when detecting at the laser frequency, hence favouring this configuration. In presence of dephasing, antibunching is spoiled and only finite values can be obtained. However, the improvement as compared to the non-corrected single-photon source is also dramatic, making this scheme of value in this case as well.

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Appendix A. Wavefunction approximation method at vanishing pumping regime

In the context of this work, the wavefunction approximations [74] consist of assuming that the state of the system composed by two fields, with annihilation operators ξ and c following either pseudo-spin or bosonic algebra, can be approximated by a pure state, which reads in the Fock state basis

$$|\psi\rangle = \sum_{n,m} \mathcal{C}_{nm} |n\rangle_c |m\rangle_\xi \equiv \sum_{n,m} \mathcal{C}_{nm} |n, m\rangle, \quad (\text{A.1})$$

where \mathcal{C}_{nm} are the probability amplitude of having n photons in the field of operator ξ and m photons in the field of operator c . The summation extends over the boundaries of the respective spaces, which is 1 for a two-level system and ∞ for a bosonic one, which in practice is truncated to N . Since the dynamics of the system is given by the master equation

$$\partial_t \rho = i[\rho, H] + \sum_k (\tilde{\Gamma}_k/2) \mathcal{L}_{j_k} \rho, \quad (\text{A.2})$$

where H is the Hamiltonian of the system and assuming the dissipation in the form of “jump operators” j_k at rates $\tilde{\Gamma}_k$, the dynamics of the wavefunction is given by Schrödinger equation

$$\partial_t |\psi\rangle = -i H_{\text{eff}} |\psi\rangle, \quad (\text{A.3})$$

where H_{eff} is a non-hermitian Hamiltonian constructed as $H_{\text{eff}} = H - i \sum_k \tilde{\Gamma}_k j_k^\dagger j_k$, and the coefficients evolve as

$$\partial_t \mathcal{C}_{nm} = -i \sum_{p,q} \langle n, m | H_{\text{eff}} | p, q \rangle \mathcal{C}_{pq}. \quad (\text{A.4})$$

In our particular case, in which we describe the excitation of a sensor (an harmonic oscillator) by the emission of a two-level system, which in turn is driven in the Heitler regime by a laser, the Hamiltonian is the one given in Eq. (5) of the main text:

$$H = \Delta_\sigma \sigma^\dagger \sigma + \Delta_a a^\dagger a + \Omega_\sigma (\sigma^\dagger + \sigma) + gt (\sigma^\dagger a + a^\dagger \sigma) - ir |\beta| (a^\dagger e^{i\phi} - a e^{-i\phi}). \quad (\text{A.5})$$

Here the two-level system is driven with intensity Ω_σ and is coupled to the cavity with strength g , the cavity is also driven by a field $\beta e^{i\phi}$ and the detuning between the two-level system (resp. sensor) and the driving laser is given by Δ_σ (resp. Δ_a). These fields are attenuated by the transmission t and reflection r coefficients of the beam splitter in which they interfere. Considering that the two-level system and the cavity have decay rates γ_σ and Γ , respectively, the effective Hamiltonian that describes the dynamics in the wavefunction approximation reads[‡]

$$H_{\text{eff}} = H - \frac{i}{2} (\gamma_\sigma \sigma^\dagger \sigma + \Gamma a^\dagger a), \quad (\text{A.6})$$

where H is the Hamiltonian in Eq. (A.5). Replacing the effective Hamiltonian in Eq. (A.6) in the expression in Eq. (A.3), we obtain the differential equations for the

[‡] The dephasing of the two-level system enters the description as an extra Lindblad term in the master equation: $(\gamma_\phi/2) \mathcal{L}_{\sigma^\dagger \sigma} \rho$, where γ_ϕ is the rate of dephasing. However, the effect of this term is the decoherence of the state of the two-level system, which affects only the off-diagonal elements of the *density matrix* of the two-level system, and thus cannot be described through a wavefunction approximation.

coefficients of interest:

$$i\partial_t \mathcal{C}_{01} = \Omega_\sigma + gt\mathcal{C}_{10} + ir|\beta|e^{-i\phi}\mathcal{C}_{11} + \left(\Delta_\sigma - i\frac{\gamma_\sigma}{2}\right)\mathcal{C}_{01}, \quad (\text{A.7})$$

$$i\partial_t \mathcal{C}_{10} = -ir|\beta|e^{i\phi} + \Omega_\sigma\mathcal{C}_{11} + gt\mathcal{C}_{01} + \sqrt{2}ir|\beta|e^{-i\phi}\mathcal{C}_{20} + \left(\Delta_a - i\frac{\Gamma}{2}\right)\mathcal{C}_{10}, \quad (\text{A.8})$$

$$i\partial_t \mathcal{C}_{11} = \Omega_\sigma\mathcal{C}_{10} - ir|\beta|e^{i\phi}\mathcal{C}_{01} + \sqrt{2}gt\mathcal{C}_{20} + \left(\Delta_\sigma + \Delta_a - i\frac{\gamma_\sigma + \Gamma}{2}\right)\mathcal{C}_{11}, \quad (\text{A.9})$$

$$i\partial_t \mathcal{C}_{20} = \sqrt{2}gt\mathcal{C}_{11} - \sqrt{2}ir|\beta|e^{i\phi}\mathcal{C}_{10} + 2\left(\Delta_a - i\frac{\Gamma}{2}\right)\mathcal{C}_{20}, \quad (\text{A.10})$$

where we have assumed that the driving to the two-level system is low enough so that the states with three or more excitations can be safely neglected, and that the driving laser is resonant to both the two-level system and the sensor. Assuming that the coherent field that drives the sensor can be written as a fraction of the field that drives the two-level system, as in Eq. (6), and to leading order in the coupling and the driving intensity of the two-level system, the solution to Eqs. (A.7-A.10) is

$$\mathcal{C}_{01} = -\frac{2i\Omega_\sigma}{\gamma_\sigma + 2i\Delta_\sigma}, \quad (\text{A.11})$$

$$\mathcal{C}_{10} = -\frac{2gt\Omega_\sigma[2\gamma_\sigma + (\gamma_\sigma + 2i\Delta_\sigma)\mathcal{F}e^{i\phi}]}{\gamma_\sigma(\Gamma + 2i\Delta_a)(\gamma_\sigma + 2i\Delta_\sigma)}, \quad (\text{A.12})$$

$$\mathcal{C}_{11} = \frac{4igt\Omega_\sigma^2[2\gamma_\sigma + (\gamma_+ + 2i\Delta_+)\mathcal{F}e^{i\phi}]}{\gamma_\sigma(\Gamma + 2i\Delta_a)(\gamma_\sigma + 2i\Delta_\sigma)(\gamma_+ + 2i\Delta_+)}, \quad (\text{A.13})$$

$$\mathcal{C}_{20} = \frac{2\sqrt{2}g^2t^2\Omega_\sigma^2\{4\gamma_\sigma^2 + (\gamma_+ + 2i\Delta_+)e^{i\phi}\mathcal{F}[4\gamma_\sigma + e^{i\phi}\mathcal{F}(\gamma_\sigma + 2i\Delta_\sigma)]\}}{\gamma_\sigma^2(\Gamma + 2i\Delta_a)^2(\gamma_\sigma + 2i\Delta_\sigma)(\gamma_+ + 2i\Delta_+)}. \quad (\text{A.14})$$

The population of both the two-level system and the cavity, as well as $g_a^{(2)}$ can be obtained from the coefficients in Eqs. (A.11-A.14) as $n_a = |\mathcal{C}_{10}|^2$, $\langle n_\sigma \rangle = |\mathcal{C}_{01}|^2$ and $g_a^{(2)} = 2|\mathcal{C}_{20}|^2/|\mathcal{C}_{10}|^4$, respectively. The cancellation of the coefficient \mathcal{C}_{20} , and therefore of $g_a^{(2)}$, yields the condition on the attenuation factor

$$\mathcal{F} = -\frac{2\gamma_\sigma e^{-i\phi}}{\gamma_\sigma + 2i\Delta_\sigma} \left(1 \pm \sqrt{\frac{\Gamma + 2i\Delta_a}{\Gamma_+ + 2i\Delta_+}}\right), \quad (\text{A.15})$$

in agreement with Eq. (8) of the main text.