Crashworthiness design and optimisation of windowed tubes under axial impact loading

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Abstract

Thin-walled structures are frequently used as energy absorbers in the automotive, railway and aviation industries. This paper addresses the crashworthiness performance of thin-walled windowed tubes under dynamic impact loading. Different shapes of cut-outs were introduced to thin-walled tubes with different cross-sectional shapes to create windowed tubes. Explicit finite element code, LS-DYNA, was used to simulate the crushing behaviour of the windowed tubes under axial impact loading. The Finite Element (FE) model was validated by conducting experimental tests and showing that the numerical and experimental responses are comparable. The crashworthiness responses of the different windowed tubes were compared and the best performing tube was identified using a multi-criteria decision-making method known as Technique of Order Preference by Similarity to Ideal Solution (TOPSIS). It was found that a circular tube with a square window shape outperforms all other sections and exhibits the best energy absorption characteristics.

Subsequently, a multi-objective optimisation analysis was performed to find the optimal configuration of the best tube. Response Surface Methodology (RSM) was used to develop
models for the energy absorption responses of the tube. The design variables were selected to describe size, number, and distributions of the windows, while specific energy absorption (SEA) and peak crush force (PCF) were set as design responses. Parametric analysis was conducted to understand the effects of the design variables on the crashworthiness behaviour and the optimal configuration was identified.

**Keywords**

Windowed tube, Energy absorption Impact, Dynamic loading, crashworthiness, optimisation

1. Introduction

Thin-walled structures are widely used in aerospace and automotive industries as energy absorption components that protect occupants during a collision event [1]. During an impact event, thin-walled tubes crush progressively dissipating the impact energy and thus reducing the influence of impact forces on the passengers. The goal for an engineer is to design a system that provides the maximum protection for the occupants against impact generated from vehicle accidents. This means that the thin-walled structures must increase the energy absorption and decrease the impact force for the safety of passengers. Thus, the design requirements for improving the crashworthiness performance of thin-walled structures are high SEA, low PCF and high Crush Force Efficiency (CFE).

The energy absorption behaviour of thin-walled structures was studied under different types of loading including axial loading [2], [3], lateral loading [4]–[10], oblique loading [11], and bending loading [12], [13].

For example, Tran and Baroutaji [11] investigated the crashworthiness behaviour of multi-cell triangular tubes under axial and oblique loading. They have employed a multi-objective optimization algorithm to find the optimal design of the tube under multiple loading cases. Fang
et al [14] used multi-objective robust design optimisation method to find the optimal crashworthiness design of foam-filled bitubal structures. Baroutaji and co-authors [5], [8] studied the crush and energy absorption behaviour of circular and oblong tubes under lateral loading and obtained the optimal design for each one of them. Among the various types of deformation modes, the axial crushing mode was identified as the most effective mode for energy absorption as most of the tube’s material deforms plastically thus making use of the majority of the tube’s mass [1]. With an aim of improving the energy absorption behaviour of thin-walled structures, tubes with unconventional shapes or materials have been adopted such as multi-cell tubes [15], [16], functionally graded thickness tubes [17], [18], foam-filling tubes [14], [19]–[22], corrugated tubes [23], grooved tubes [24], [25], and windowed tubes [26]–[28]. Eyvazian et al [23] investigated experimentally and theoretically the axial crushing behaviour of tubes with shallow and deep corrugations. Their results showed that corrugated tubes demonstrate excellent energy absorption characteristics in terms of more uniform force-displacement curve, lower of initial peak load, and controllable failure mechanism.

Embedding cut-outs or windows on the walls of thin-walled tubes were introduced as a mean of reducing the initial peak force and improving the overall crashworthiness behaviour of the tubes [28]–[33]. Gupta et al [34], [35] studied the crush response of circular tubes which contain circular holes on their walls. They found that the holes reduced the initial impact force and helped in avoiding the overall buckling when the slenderness ratio (length to diameter) was large. A confirmation of the advantages of the windowed tubes was presented by Song et al [36]–[38] who studied the crashworthiness behaviour of thin-walled square tubes with patterned windows. The authors found that by introducing windows on the tube wall, the energy absorption
characteristics such as initial peak force and energy absorption capacity could be significantly improved. Also, H. Nikkhah et al [26] investigated the effect of the hole’s shape on the crashworthiness of square tubes under dynamic loading condition. Their research showed that rectangular and square-shaped holes provide the best crashworthiness performance for the square tubes.

Despite the excellent crashworthiness performance of the windowed tubes, they only received limited interest from researchers. No attempt was made in the literature to compare the crashworthiness behaviour of windowed tubes with different cross-sectional shape for both the tubes and the windows. Thus, in this study, the crush behaviour and energy absorption capability of a new set of windowed tubes are investigated, compared, and optimised.

2. Material and Methods

2.1 Geometrical description of windowed tubes

Tubes with three different cross-sectional shapes including circular, hexagonal and square as shown in Figure 1, were tested in this study. All these tubes have the same cross-section area of 1225 mm², a length of 90 mm, and a wall thickness of 2 mm. Three shapes of holes namely circular, hexagonal and square were introduced to the previous tubes to create windowed tubes. The holes, i.e. windows, have the same cross section area of 78.5 mm² and distributed equally along the length of the tubes as shown in Figure 2. The distance between the top (bottom) of the tube is 10 mm and the distance between holes is 23.35mm.

2.2 Finite Element Modelling

Explicit Finite Element (FE) software package, LS-DYNA, was adopted for simulating the axial crushing of the windowed tubes. The FE model is consisted of three main parts including, the upper moving base, the thin-walled tube, and the lower stationary base, as shown in Figure 3.
The upper base is modelled as a rigid body and constrained to travel along the axial direction of the tube. The bottom base was also modelled as a rigid body where its movement was constrained in all directions so it became stationary. The bottom end of the tube is fixed onto the bottom base while the top of the tube is free. The rigid upper base, with a mass of 500 kg and moves at a velocity of 15 m/sec, impacts the tube at its top end. The mass was chosen to ensure full deformation of the tube while the impact speed is a typical value used for the automobile crashworthiness applications [39]. A Belytschko-Tsay 4-node shell element with 5 integration points through the thickness is used to mesh the model because it is suitable for the large deformation of thin-walled tubes [26]. A mesh sensitivity analysis was performed to determine the optimal mesh size of the finite element model as shown in Figure 4. An element size of 1 mm, which results in a total number of elements of 50000, was found to provide a good convergence for both crashworthiness responses within reasonable solution time and thus it was used throughout this study. Automatic single surface and surface to surface contacts with a friction coefficient of 0.15 were used to represent the tubes self-contact and tube-to-wall contact, respectively. The tubes were made of aluminium alloy AA6060-T4 with a density of 2700kg/m³, Young's modulus of 68 GPa, and Poisson's ratio of 0.33. Tensile tests were performed using a universal test machine to obtain the true stress-strain curve of the material, as shown in Figure 5. The Mat_Piecewise_Linear_Plasticity model #24 in LS-DYNA was used to represent the material behaviour of the aluminium alloy AA6060 T4 during the crushing process. Since aluminium alloy is insensitive to strain rate under dynamic load, the material strain rate effects were ignored in this study [40].

2.3 Multi-objective optimisation
Formulation of optimisation problem

A multi-objective optimisation problem is given by a general mathematical formulation as shown in equation 1:

\[
\begin{align*}
\text{Minimise} & \quad f(x) = [f_1(x), f_2(x), \ldots, f_i(x)] \\
\text{s. t} & \quad x^l \leq x \leq x^u
\end{align*}
\]

1

Where \( x=(x_1, x_2, \ldots, x_k) \) is the vector of \( k \) design variables, \( x_l \) and \( x_u \) are respectively the lower and upper bounds of the design variables, \( f(x) \) are the objective functions. Among the various crashworthiness characteristics, SEA and PCF, as shown in Figure 6, were selected as design responses in this study.

A thin-walled energy absorber should absorb the maximum possible amount of energy per unit mass, in order to allow for a lightweight design with efficient fuel consumption and thus the SEA should be maximised. Also, in order to avoid the severe injury and reduce the jerking effects felt by the occupants in the survival space, the crushing of the energy absorber should not lead to high decelerations and this can be achieved by minimising PCF. Factors describing the size, number, and distributions of windows including the characteristic dimension of the holes (\( d \)), the number of holes in the horizontal direction (\( N_h \)), and the number of holes in the vertical direction (\( N_v \)) were selected as design variables.

Response Surface Methodology (RSM) and optimisation algorithm

RSM is a set of mathematical and statistical techniques that can be used for modelling the responses of a specific engineering system as functions of the controllable input variables. Generally, the Response Surface (RS) model can be expressed by equation 2

\[ Y = f (x_1, x_2, \ldots, x_k) \]

2

Where: \( k \) is the number of independent variables
If a second order polynomial function is used in RSM to describe the relationship between the responses and the independent variables then the RS model is given as in equation 3

\[ y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ii} x_i^2 + \varepsilon \]

Where \( b_0, b_i, b_{ij}, b_{ii} \) known as regression coefficients of the model, \( \varepsilon \) is unobserved random error.

In the current study, RSM was used to construct models which relate the crashworthiness responses, i.e. SEA and PCF, to different design variables. The advantage of employing RSM is that the crashworthiness responses in a particular design space can be identified through performing a reduced number of experiments or simulations at sampling design points. The sampling design points can be generated using different methods offered by Design of Experiment (DoE) such as factorial, Box–Behnken design (BBD), central-composite design (CCD) and D-optimal. The adequacy of the developed Response Surface (RS) models is checked via the analysis of variance (ANOVA) to confirm their capability in predicting the crashworthiness responses accurately. Various statistical measures, R-square parameter, Adjusted R-square, and Adeq Precision, were used to confirm the statistical significance of the models. Once the RS models have been developed and tested for adequacy, they can be used in the multi-objective optimisation formula. Desirability approach was used to solve the multi-objective crashworthiness optimisation problem as denoted in equation 1. This technique has received considerable attention for solving the multi-objective crashworthiness optimisation problems due to its simplicity, low computational cost and rapid convergence [5], [7], [8], [41].

In this approach, all multiple crashworthiness responses, i.e. SEA and PCF, are combined into a single non-dimensional objective function, called an overall desirability function, which only
generates one solution for the optimisation problem. The main steps for using the desirability
approach in the multi-objective optimisation problem are detailed in Figure 7.

2.4 TOPSIS method

TOPSIS method is used in this study to identify the tube with best crashworthiness performance
in terms of both SEA and PCF responses. TOPSIS, an abbreviation of Technique of Order
Preference by Similarity to Ideal Solution, is a multiple criteria decision-making method which
allows determining the best candidate among others based on multiple responses [42], [43]. It
has received increased attention for crashworthiness investigations [26], [44]. In this method, it is
assumed that among the different candidates there is positive ideal candidate and negative ideal
candidate where the former has the best level for all attributes considered and the latter has the
worst level. A candidate gets higher score if it is closer to the positive ideal candidate and further
from the negative ideal candidate [43].

TOPSIS can be implemented in the current design problem as follows;

Firstly a design matrix (X), which is consisted of m candidates and n criteria, should be
constructed. This design matrix maps the candidates, i.e. different windowed tubes, to the
selection criteria, i.e. crashworthiness responses including SEA and PCF. Each element of this
matrix (xij) is the value of candidate i with respect to criterion j.

Secondly, the design matrix is normalised as shown in equation 4

\[ r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}} \]  \hspace{1cm} (4)

In which \( r_{ij} \) is the element of normalised design matrix R.

Thirdly, each criterion is assigned a weight factor. In the present study, all criteria have the same
weight and thus this step has no effect on the final score of each tube. 

Fourthly, from the matrix R, both positive ideal and negative ideal candidates can be determined. Positive ideal candidate \( (A^+) \) has lowest PCF and highest SEA while negative ideal candidate \( (A^-) \) has highest PCF and lowest SEA among all candidates. The distance of the \( i \)th candidate to the ideal positive and negative candidates, denoted by \( D_i^+ \) or \( D_i^- \), is given by equation 5

\[
D_i^{+/−} = \sqrt{\sum_{j=1}^{n} (r_{ij} - A_j^{+/−})^2}
\]

The relative closeness of each candidate to the ideal candidates is calculated by the following equation 6

\[
S_i^+ = \frac{D_i^-}{D_i^+ + D_i^-}
\]

In which \( S_i^+ \) is the relative closeness of the \( i \)th candidate. The candidate with \( S^+ \) closer to 1 is the best candidate.

3. Results and discussion

3.1 Experimental validation of FE model

To validate the FE model, axial crushing tests were conducted on different configurations of simple and windowed tubes. The geometrical configurations of the tubes used in the experiments along with the experimental set-up are shown in Figure 8. The tests were performed using a universal test machine (type STM-150) where a sample was placed between the bottom stationary and the top moving bases. The crushing process was quasi-static with a crushing speed of 10 mm/min. Figure 9 shows the crushing response and deformation modes obtained from experiments and simulations. It can be seen that the finite element models of the different tubes provide excellent predictions for force-displacement curves as well as the deformation modes of
the tubes. As a result, the FE model in this paper can be considered as an accurate model and can be extended to model the other simple and windowed tubes.

3.2 Crush analysis of windowed tubes with different geometrical shape

In this section, the crushing performance of simple and windowed tubes with different geometrical shapes is investigated. SEA and PCF responses were used to evaluate and compare the crashworthiness behaviour of the different tubes. The force-displacement curves, SEA and PCF responses, and deformation modes of all tubes are shown in Figure 10 and Figure 12, respectively. Generally, the windowed tubes allow for using longer tubes without undergoing the global bending mode [34]. However, for the current study, both simple and windowed tubes have the same length and they are all deformed progressively where none of them undergoes the inefficient global bending mode. Furthermore, it can be seen from the deformation modes, Figure 12, that the simple tubes have a more desirable symmetrical deformation mode with shorter wavelengths and greater number of folds than those observed in the windowed tubes. This is due to the fact that the simple tubes have a uniform material distribution at the deformation locations and this makes the axial stiffness of these tubes more uniform than the windowed tubes, which have material discontinuities, leading to a more regular deformation pattern. The deformation modes of circular windowed tubes, as shown in Figure 12, are irregular and their crush response, as depicted in Figure 10 (a), are quite different from each other. This indicates that the collapse behaviour of such structures is sensitive to the window profile. On the other hand, the crushing behaviour of square and hexagonal windowed tubes was less sensitive to the change in the window profile where the force-displacement responses of these structures, as shown in Figure 10 (b and c), exhibit slight difference when changing the geometrical shape of the windows. Also, both simple and windowed tubes exhibit similar trend for force-displacement response in
which the crushing forces increase sharply to initial peak values and then start fluctuating periodically around a mean value in a fashion corresponding to the formation of folds during the crushing. However, the windowed tubes offer lower PCF and also lower fluctuation in the post-collapse stages. With regards to SEA and PCF responses, simple tubes show higher SEA and PCF for all cross-section shapes. This is due to the fact that the plastic deformation in the windowed tubes is initiated at the windows region which has less material and thus it requires less force to initiate the collapse and absorbs lower energy compared to the simple tubes.

3.3 Identifying the best windowed tube using TOPSIS

TOPSIS method, which is a multi-criteria decision-making method, was used with SEA and PCF responses to determine the best performing windowed tube. In the present study, there are 12 candidates, as shown in Figure 2, and each one has two crashworthiness metrics, i.e. SEA and PCF, so the total number of criteria is 24, as shown in Table 1. The final score and ranking of each tube obtained by the application of TOPSIS method are presented in Table 2. It can be seen that circular tube with square window (C-S) is the best in the overall performance while the simple square tube (S) has the lowest ranking.

3.4 Parametric analysis and optimisation of the best windowed tube

Response Surface (RS) models of SEA and PCF Central-composite design (CCD) was adopted to create the sampling design points in this study. CCD is highly efficient sampling strategy that allows for creating accurate RS models with lower number of experiments. The geometrical parameters of the created windows including the width of the window (d), the number of windows in the horizontal direction (N_h), and the number of windows in the vertical direction (N_v) were selected as design variables. The upper and lower limits of the geometrical parameters for all sampling points are illustrated in Table 3. FE models were created for the C-S windowed tubes with the geometrical parameters corresponding to the
sampling design points, as shown in Figure 13, and the crashworthiness responses were determined. The different combinations of the design variables with corresponding design responses are tabulated in Table 4. The RS models of the SEA and PCF in terms of the geometrical factors of the holes are shown in equations 7 and 8, respectively

\[
(SEA) = 14984.6 - 1061.62 \times d + 514.3 \times N_h + 627.7 \times N_v - 263.33 \times N_h \times N_v \\
(PCA) = 1.035 + 1.14 \times 10^{-4} \times d + 5.97 \times 10^{-4} \times N_h - 1.13 \times 10^{-4} \times N_v - 1.78 \times 10^{-4} \times d \times N_h
\]

The accuracy of the aforementioned RS models was checked using Analysis of Variance (ANOVA) statistical technique. Table 5 summarises the statistical measurements for the developed RS models for SEA and PCF. As it can be seen that all models show high F-value and very low P-value which confirm that these models are significant. Additionally, both models exhibit high enough values of adequate precision ratios which are greater than 4 and this means that the models have insignificant noise. Additionally, the RS models show high values of R-squared (coefficient of determination) and they also exhibit a very good agreement between the predicted and adjusted R-squared values. The relationship between the actual responses, obtained from the simulations, and those predicted by the developed RS models are shown in Figure 14. It is clear that the residuals are close to the diagonal line which also confirms that the predictive capabilities of the developed RS models are very good. All of the aforementioned measurements prove that the developed RS models are accurate and valid and thus they can be used to navigate the whole design space.
Parametric analysis of the C-S windowed tube

The crashworthiness responses of windowed C-S tube depend strongly on the windows parameters including the size, number and distribution. Thus, parametric analysis of these responses was conducted using the developed RS models in the previous section.

3.4.2.1 Effect of design variables on SEA

Figure 15 demonstrates the changes of SEA with d and $N_v$ for two different values of $N_h$. It can be seen that the SEA tends to increase with decreasing all design variables, i.e. d, $N_v$, and $N_h$. The energy absorption capacity under axial loading is dominated by the amount of material available to be plastically deformed [1]. Introducing the windows on the side walls of the tube reduces the amount of the material at the deformation regions and this reduces the energy absorption capacity. The larger the windows are, the less the material that can undergo plastic deformation is. Thus, decreasing the number and size of windows means that there is more material in the tube to be deformed plastically and participate in the energy absorption process. By inspecting the figure closely, it can be seen that the influence of $N_v$ on SEA response is almost insignificant when the $N_h$ is small and it becomes more obvious by increasing the number of windows in the horizontal direction. This is generally due to the effect of interaction between $N_h$ and $N_v$ on the SEA which makes the rate of change of SEA with $N_v$ depends strongly on the setting of $N_h$. Among all design variables, the window size, d, is the most dominant variable to influence of the SEA response. For a tube with maximum number of holes in both horizontal and vertical directions, increasing d from 4 mm to 8 mm causes a decrease of 19% in the SEA. It is clear that a tube with d of 8 mm, $N_v$ of 5, and $N_h$ of 6 absorb the lowest amount of energy per unit mass.
3.4.2.2 Effect of design variables on PCF

Figure 16 illustrates the variation of PCF with d and N_v for N_h of 2 and 6, respectively. It is evident from these plots that PCF decreases when the size and number of windows increases. Generally, a C-S tube with smaller size and less number of windows contains more material and thus it requires a greater magnitude of force to initiate the crushing process. Obviously, a C-S tube with d of 4 mm, N_v of 1, and N_h of 2 has the highest PCF. Despite the C-S tubes with higher number and/or bigger size of windows may absorb lower SEA, they would generate lower PCF during their crushing and this can be seen as an advantage for the safety of the occupants. A C-S tube with smaller size and lower number of windows produces a PCF which is 37% less than its counterpart with the highest number and size of windows. Meanwhile, the SEA drops only by 26% when using a tube with more windows. Concerning the PCF response, the window size seems to be the most influencing factor while N_h and N_v are the second and least influencing factors, respectively. This can be explained by the fact that the plastic deformation begins with formation one fold at either location of the windows so the bigger the windows are, the less the material is left at the region and hence a lower PCF is needed to initiate the crushing. Since PCF only corresponds to a formation of the first fold during the deformation process, the influence of the number of windows on PCF is insignificant. For example, when N_v and N_h set at maximum values, increasing the window size from 4 mm to 8 mm decreases the magnitude of PCF from 30.28 kN to 20.92 kN; which account for a 31% decrease in PCF. Similarly, for maximum values of d and N_h, varying the N_v from maximum to minimum values only change PCF by 4.4%. 
3.4.2.3 Effect of design variables on deformation modes

Figure 17 exhibits the deformation mode of C-S tubes representing the design points. It is clear that all C-S tubes have maintained a progressive collapse mode and none of them has undergone the inefficient global bending one. The sample C-S-7 which has a moderate number of windows with small size seems to have the best crushing mode with shorter wavelength and greater number of folds. This trend might be due to the fact that the small size windows are more efficient in redistributing the axial load which in turn makes the stiffness of the tube more uniform leading to a more regular deformation mode. Samples with bigger window size such as (C-S=1, C-S-3, C-S-6, and C-S-13) tend to develop lower number of folds during the crushing. This is because the windows with bigger size promote the creation of folds with greater wavelength and this, in turn, reduces the number of folds generated during the total deformation process. Additionally, it is evident from the deformation modes reported in Figure 17 that the tubes with bigger windows have irregular deformation modes and this is consistent with the findings of other studies [36]. Another observation that can be reported from Figure 17 is that the moderate number of windows in the vertical direction seems to promote a better deformation mode with higher number of folds as it can be seen in C-S-4, C-S-5, C-S-7, and C-S-12.

Multi-objective optimisation results

The final multi-objective crashworthiness optimisation formula of the windowed tubes in terms of SEA and PCF as design responses, and d, \( N_h \), and \( N_v \) as design variables can be expressed as in equation 9:

\[
\begin{align*}
\text{Maxm} \text{ise} & \quad \text{SEA} = f_1(d, N_h, N_v) \\
\text{Minm} \text{ise} & \quad \text{PCF} = f_2(d, N_h, N_v) \\
\text{s.t} & \quad 4 \leq d \leq 8 \\
\text{s.t} & \quad 2 \leq N_h \leq 6 \\
\text{s.t} & \quad 1 \leq N_v \leq 5
\end{align*}
\]
The desirability approach was used to solve the above equation. Figure 18 shows a contour plot of the desirability objective as a function of \( d \) and \( N_v \) at upper and lower limits of \( N_h \). It is clear that desirability increases as \( d \) and \( N_h \) increases while \( N_v \) decreases. The greatest desirability was obtained in a tube with \( d \) of 8 mm, \( N_h \) of 6, and \( N_v \) of 1. Thus, the one can conclude that the optimal shape of the windowed tube can be obtained by increasing \( d \), increasing the number of holes in the horizontal direction and decreasing the number of holes in the vertical direction.

To validate the optimisation results, the crashworthiness responses of the optimal tube were obtained by constructing a FE model. The force-displacement response of the optimal tube, as well as a comparison between the numerical results and RS results, are shown in Figure 19. It is clear that the numerical results are in excellent agreement with those obtained from the optimisation algorithm which inferno the validity of the optimised results.

4. Conclusion

In this paper, cut-outs with different shapes were introduced to thin-walled structures with different cross-sectional profiles including circular, square, and hexagonal to create windowed tubes. The crushing and energy absorption behaviour of these tubes were studied under axial dynamic loading via experimental tests and numerical simulations. The numerical models were created using LS-DYNA and validated with experimental tests. The crashworthiness responses of the different simple and windowed tubes were computed. The results revealed that the windowed tubes exhibit less PCF than the simple tubes and this was considered as an advantage for the windowed tubes over their simple counterparts. However, the simple tubes showed higher SEA and better deformation mode than the windowed tubes. The performance of the different tubes was compared and ranked using TOPSIS with PCF and SEA as design criteria. The circular tube with square windows has the highest score and was selected as the best tube in the
study with better combined SEA and PCF responses. Finally, RSM and desirability approach were used to analyse and optimise the best performing tube. It was found that the optimal tube is the one that has bigger window size, higher number of windows in the horizontal directions but lower number of holes in the vertical direction.

5. References


<table>
<thead>
<tr>
<th>Tube</th>
<th><strong>PCF (kN)</strong></th>
<th><strong>SEA (kJ/kg)</strong></th>
<th>Tube</th>
<th><strong>PCF (kN)</strong></th>
<th><strong>SEA (kJ/kg)</strong></th>
<th>Tube</th>
<th><strong>PCF (kN)</strong></th>
<th><strong>SEA (kJ/kg)</strong></th>
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<tr>
<td>C</td>
<td>37.4</td>
<td>23.0</td>
<td>H</td>
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<td>21.5</td>
<td>S</td>
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<td>18.2</td>
<td>H-C</td>
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<td>S-C</td>
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<td>H-H</td>
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Table 2: Score and rank of all tubes obtained by TOPSIS method

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<th>Tube</th>
<th>Score</th>
<th>Rank</th>
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<tr>
<td>C</td>
<td>0.0733</td>
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</tr>
<tr>
<td>C-C</td>
<td>0.1018</td>
<td>6</td>
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<tr>
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</tr>
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<td>H</td>
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<tr>
<td>S</td>
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Table 3: Upper and lower bounds of the design variables

<table>
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<th>Design variable</th>
<th>Lower limit</th>
<th>Upper limit</th>
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<tbody>
<tr>
<td>d</td>
<td>4 mm</td>
<td>8 mm</td>
</tr>
<tr>
<td>(N_h)</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>(N_v)</td>
<td>1</td>
<td>5</td>
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Table 4: Design matrix

<table>
<thead>
<tr>
<th>Tube</th>
<th>d (mm)</th>
<th>$N_h$</th>
<th>$N_v$</th>
<th>SEA (kJ/kg)</th>
<th>PCF (kN)</th>
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</thead>
<tbody>
<tr>
<td>C-S-1</td>
<td>8</td>
<td>2</td>
<td>5</td>
<td>19.8618</td>
<td>28.4067</td>
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<tr>
<td>C-S-2</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>23.186</td>
<td>31.5129</td>
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<tr>
<td>C-S-3</td>
<td>8</td>
<td>6</td>
<td>1</td>
<td>20.355</td>
<td>22.285</td>
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<tr>
<td>C-S-4</td>
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<td>6</td>
<td>3</td>
<td>20.8589</td>
<td>24.4002</td>
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<tr>
<td>C-S-5</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>21.109</td>
<td>27.9502</td>
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<tr>
<td>C-S-6</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>16.2499</td>
<td>20.7521</td>
</tr>
<tr>
<td>C-S-7</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>21.9073</td>
<td>32.6355</td>
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<tr>
<td>C-S-8</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>23.1407</td>
<td>31.4422</td>
</tr>
<tr>
<td>C-S-9</td>
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<td>6</td>
<td>5</td>
<td>21.0532</td>
<td>29.8993</td>
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<td>C-S-10</td>
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<td>6</td>
<td>1</td>
<td>23.1846</td>
<td>31.7561</td>
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<td>1</td>
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<td>3</td>
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<td>1</td>
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<td>C-S-15</td>
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<td>2</td>
<td>3</td>
<td>21.4</td>
<td>32.1318</td>
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Table 5: Summary of ANOVA analysis for the developed RS of SEA and PCF responses

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<th>Model</th>
<th>F-Value</th>
<th>P-Value</th>
<th>Statistical measurements</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>R²</td>
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<tr>
<td>$SEA = f(d, N_h, N_v)$</td>
<td>27.52</td>
<td>&lt;0.0001</td>
<td>0.91</td>
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<tr>
<td>$PCF = f(d, N_h, N_v)$</td>
<td>52.4</td>
<td>&lt;0.0001</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Figure 1: Different cross-section

a) Left to right (C, C-C, C-H, C-S)  
b) Left to right (H, H-C, H-H, H-S)  
c) Left to right (S, S-C, S-H, S-S)

Figure 2: Windowed tubes: a) Circular, b) Hexagonal, c) Square
Figure 3: Schematic of finite element model of H tube under axial loading
Figure 4: Mesh convergence of (a) SEA response, and (b) PCF response for a simple square tube
Figure 5: Tensile test: a) True stress-strain of AL6060-T4, b) Universal test machine (STM-150)
Figure 6: Typical crush force-displacement response under axial loading with explanation of SEA and PCF responses

Absorbed Energy (E) \[ E = \int_0^{\delta} F(\delta) \cdot d\delta \]

Specific Energy Absorption (SEA) \[ \text{SEA} = \frac{E}{m} \]
Figure 7: Flow chart of optimisation scheme using the desirability approach
Figure 8: (a) Geometrical details of samples used for FEM validation (b) Experimental test setup
Figure 9: Comparison of experimental and simulation results: (a) Simple square tube (b) Windowed square tube (c) Windowed circular tube
Figure 10: Force-displacement curves: a) Circular tube, b) Hexagonal tube, c) Square tube
Figure 11: Crashworthiness responses for all tube a) SEA, b) PCF
Figure 12: Deformation modes of simple and windowed tubes
Figure 13: Geometry of C-S windowed tubes used to construct RS models

(SEA)

(PCF)

Figure 14: Comparison of FE simulations values against predicted values obtained from RS models at the design points
Figure 15: The influence of window parameters on SEA
Figure 16: The influence of window parameters on PCF
Figure 17: Deformation mode of circular windowed tubes
Figure 18: Variation of desirability objective with design variables
Figure 19: Numerical results of the optimal tube

<table>
<thead>
<tr>
<th></th>
<th>FE</th>
<th>RS</th>
<th>RE%</th>
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</thead>
<tbody>
<tr>
<td>SEA (kJ/kg)</td>
<td>21.63144</td>
<td>20.49179</td>
<td>5.268486</td>
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<tr>
<td>PCF (kN)</td>
<td>21.75</td>
<td>21.86948</td>
<td>0.549316</td>
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