Methods of Analysing Ordinal/Interval Questionnaire Data using Fuzzy Mathematical Principle

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Abstract

Two methods of analysing interval/ordinal questionnaire data based upon principles from fuzzy mathematics and artificial intelligence are described.

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Methods of Analysing Ordinal/Interval Questionnaire Data Using Fuzzy Mathematical Principle

Research questionnaires in which respondents are asked to make judgements against a linguistically constructed ordinal scale or an interval scale with referent statements are frequently employed in various areas of management, marketing and behavioural research. Statistical methods for the treatment of the results of such questionnaires may be limited, leading to the need to develop alternative ways of analysing and computing results.

The desire to develop a computable model based upon judgements made by various individuals expressed within an ordinal/interval scale leads to the consideration of some well developed principles of fuzzy sets and arithmetic.

Questionnaire data

Ordinal or interval scales of various sorts are quite frequently employed to derive judgements from a sample population so that for example respondents may be asked to judge whether they agree with a particular statement such as..

"the information systems is …"
1. Essential
2. Very useful
3. Of some use
4. Of little use
5. Of no use

The treatment of the results of such a survey must be dependent upon the meaning which is ascribed to the scale and the degree of statistical rigor which it is desired to apply. Pervan and Class in 'The Use and Misuse of Statistical Methods in Information Systems Research' (Pervan & Class, 1992) address this important issue by means of a discussion of various applications of these types of scales. Pervan and Class discuss three sorts of scale, Nominal, Ordinal and Interval.

Nominal scales are simply lists of mutually exclusive categories. So, for example, if the answer to a question is a simple *yes* or *no* this is a nominal scale.

Ordinal scales imply a ranking of various alternatives, thus in the example above, *Very useful* implies a higher value than *Of some use*. The relative position in a ordinal scale is important but there is no implication that *Very useful* is in some precisely measurable degree greater than *Of some use*.

By contrast with an ordinal scale a true interval scale implies an equality of interval between the various points on the scale. In this case, applying to the example, we would have to be able to map *Very useful* in some precise way to the other points on the scale so that we could be in position to say precisely how it related to any other position; for example that *Very useful* meant exactly twice as useful as *Of some use*.

Pervan and Class point out that there is a frequent confusion of ordinal scales with true interval scales depending upon the use to which the scale is being put. In particular in certain uses of ordinal scales for example;

> the rating of a characteristic ; This use is the most controversial because it interprets ordinal measure with interval characteristics as well. Here the researcher assigns numbers to reflect relative ratings of a series of statements, then uses these numbers to interpret relative differences. (Pervan & Class, 1992 p. 212).

This is a very easy error to fall in to when numbers are derived from survey data. In fact the ability to apply meaningful statistical tests to survey results is dependent upon the nature of the scales used to derive the results. Strictly, as Pervan and Class illustrate, the valid use of (simple) statistical tests on an ordinal scale is limited to a Kolmogorov-Smirnov test that the distribution of results matches some expected value. Chi-square can be conducted on an nominal scale for the same purposes but use of t-test or other tests based upon the Normal distribution can only be used where an interval scale is being employed (implying an underlying continuous random variable).

So, in the case of the judgements of usage of (e.g.) co-operative information systems technologies discussed in (Hassall, 1998 & 1999), we are (strictly) limited in terms of the statistical analysis of results if we proceed with what is essentially an ordinal scale.

Fuzzy analysis of questionnaire data

A different perspective and a way of analysing ordinal and interval scales produced from survey questionnaires may be developed through consideration of various principles of fuzzy logic, fuzzy set theory and fuzzy arithmetic linked to the idea of *linguistic variables*. An introduction to the topics of fuzzy set theory and fuzzy arithmetic is included in *Fuzzy Sets, Fuzzy Logic, Fuzzy Methods - with Applications* (Bademar & Gottwald, 1995) and much of the following discussion is derived from this text.

A *fuzzy* set is defined by comparison with a *crisp* set as one where the membership function for the set can return a fractional value rather than the more usual 0 (definitely not a member of the set) or 1 (definitely a member of the set). If, for example, the value of the membership function is considered to represents some measure of the *"grade of membership"* (Krause & Clark, 1993, p120) of a particular hypothesis, it may be treated as a way of looking at the world which reflects judgements, such as "High Temperature" which are imprecise or vague. Various persons may judge "High Temperature" , or "Middle Age" in different ways so that any particular temperature or age may be assigned a degree of membership of the concept. Krause and Clark offer a definition for a fuzzy set (after Zadeh, 1965) who developed much of the basic theory of fuzzy sets and fuzzy logic.

> Let Ω be a frame of discernment (set of all possible values x for an attribute). Then a fuzzy set A Ω is characterised by a membership function U_A: $\Omega \equiv [0,1]$. The value $U_A(x)$ for x ♦ Ω represents the "grade of membership" of x in A. The characteristic function U_A can be thought of as a measure of the degree of compatibility of x with the concept A. (Krause & Clark, 1993 p. 120).

An important point to note at this stage is that in defining an ordinal scale we have implicitly established a set of imprecise (or vague) referents which may represent concepts to which varying degrees of membership may be assigned Thus, when a respondent makes a judgement that "the information systems is …. *Of little use*" they are doing something similar to judging that the "temperature is high". So we can take as a possible departure point that our ordinal/interval scale can be said to represent a group of possible concepts (or indeed hypotheses) and that the particular instance under consideration may belong in varying degrees to any or all of them. In practice however, because the scale is ordinal, a stronger degree of compatibility with for example the *Very useful* hypothesis than the with the *Of some use* hypothesis will, quite logically, imply an even lower degree of compatibility with the *Of little use* hypothesis.

The concept and definition of a fuzzy set leads on quite naturally to that of a fuzzy number which is spread-out, or vaguely defined version of an ordinary number. So, for example, we could consider a region around the number 3 as representing the fuzzy concept of *3 ness*. As we move away from 3 in the negative direction we will eventually reach a value which has no grade (or degree) of *3 ness* associated with it, similarly in the positive direction.

Where precisely the upper and lower bounds of *3 ness* lie, and what shape the membership function takes will depend upon the application being attempted but, in practice, we might wish to adopt a representation which will enable meaningful calculations to be made upon our numbers. Bademar and Gottwald discuss a range of possibilities including trapezoidal numbers, where the value of the membership function rises steadily from some minimum to reach a plateau of 1.0 then declines steadily to 0 again (Bademar & Gottwald, 1995 p. 56). Of more immediate potential use are so-called triangular fuzzy numbers which can be defined by three values; a minimum at which the membership function is 0, a kernel at which it is 1.0 and an maximum at which it returns to 0.

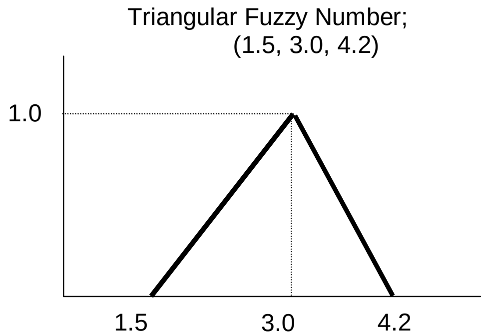

Figure 1. Triangular fuzzy number.

Arithmetic may be defined for such numbers as described below.

For triangular fuzzy number A (TFN A) having minimum value A1, kernel value A2 and maximum value A3 we write the number as (A1. A2, A3).

Let TFN A := (A1, A2, A3)

Let TFN B := (B1, B2, B3)

The sum of A and B is (A1+B1, A2+B2, A3+B3)

The difference of A and B is (A1-B3, A2-B2, A3-B1)

The sum and differences of TFNs are thus linear operations which yield other TFNs but products and quotients need not necessarily do so. Nonetheless, approximation of sums and quotients to TFNs is in practice desirable and Bademar and Gottwald report the following approximations for these operations which hold for positive TFNs.

The approximate product of A and B is (A1xB1, A2xB2, A3xB3)….. and the approximate quotient (A/B) is (A1/B3, A2/B2, A3/B1) for A1 and B1 greater than or equal to 0 (in which case A2,B2,A3 and B3 are, by definition greater than 0 and the whole fuzzy interval lies in the range >= 0 (Bademar & Gottwald, 1995 p. 56).

In terms of evaluating a judgement, or a value that has a certain vagueness associated with it, fuzzy numbers have some attractiveness and the nature of computation with imprecise values is dealt with extensively in *Representing Uncertain Knowledge* (Krause & Clark, 1993). Krause and Clark characterise the fuzzy interval defined by a particular fuzzy number as a *possibility* space, with the point at which the membership function becomes 1.0, the kernel for example of a TFN, representing a value that *necessarily* matches the concept or hypothesis being evaluated. What is becoming clear is that there can be some mapping between a value on a scale and the linguistic conventions of an ordinal/interval scale. Thus, rather than making the intervals explicitly equal because based upon some underlying physical and continuous quantity, we set them as equal by definition and then seek to interpret the results in terms of the numbers that emerge. To put it another way, and referring back to the earlier example, we assign *Essential* the value 1 and *Of no use* the value of 5 by definition.

The next development is to develop a treatment of the results of the respondent scores of such a questionnaire which recognises the inherently imprecise nature of the judgements being made. Two formulations are proposed.

Formulation 1 - triangular fuzzy numbers

Firstly we note that respondents have to choose between a series of statements on the ordinal/interval scale which one they judge most appropriate and it is argued that the choice of score is, in effect, a judgement between 3 indicator statements. Thus, as an example, respondents scores for whether a particular technology is employed may be recorded on the following scale.

1. Never employed
2. Seldom employed
3. Sometimes employed
4. Frequently employed
5. Almost always employed
6. Indispensable to task

In this interpretation, a respondent who judges 4 to be the appropriate score makes a constrained choice in the range where 3 is the minimum value and 5 the maximum. (To think of it another way, the respondent must consider which of the three hypotheses, *Sometimes employed*, *Frequently employed* and *Almost always employed* best represents their judgement of the situation.)

In the method of extracting fuzzy scores the score 4 corresponds to a triangular fuzzy number (3,4,5). Similarly, score 5 corresponds to (4,5,6), and so on. The full scoring correspondence is taken to be as follows.

1 = TFN (1,1,2)
2 = TFN (1,2,3)
3 = TFN (2,3,4)
4 = TFN (3,4,5)
5 = TFN (4,5,6)
6 = TFN (5,6,6)

The two extreme scores reflect the fact that the respondent is constrained within the range and cannot for example award a score of 7 or (say) 0.5.

With a results table of the following from

Total respondents 45						
Scores	**1**	**2**	**3**	**4**	**5**	**6**
Frequencies	7	5	11	12	7	3

Figure 2. Sample frequency table.

Taking the average weighted score for each TFN representing the appropriate score yields the TFN (2.51,3.36,4.29) when carried out with appropriate attention to arithmetic rules for TFNs thus;

(1,1,2)x7+(1,2,3)x5+(2,3,4)x11+(3,4,5)x12+(4,5,6)x7+(5,6,6)x3)/45 = (2.51,3.36,4.29)

It should be noted that, in this formulation, the kernel value is identical to the weighted average (or mean) score recorded.

Having derived the TFN this can be interpreted with reference to the original ordinal/interval scale. For example it can be noted that the kernel value is "somewhat better than sometimes employed" and that this may be taken as the 'most likely' value within the fuzzy interval.

In the formulation just discussed, understanding of the interpretation to be put upon the fuzzy triangular numbers derived is bound to be dependent upon prior experience and an degree of familiarity with the concepts. In some senses TFNs, because they present a rather clean model of a fuzzy interval, may not allow the person interpreting the results sufficient feel for the fuzziness of the linguistic space in which a judgement is being made. In these circumstances a variety of visual presentations may be preferable and a number of possibilities are offered.

Fuzzy triangular numbers interpretation

Because of the underlying linguistic nature of the ordinal/interval scale we can potentially provide a direct interpretation of the triangular fuzzy score by mapping against the scale in a way that takes into account the linguistic values assigned to each point. The basic idea is illustrated in the diagram below.

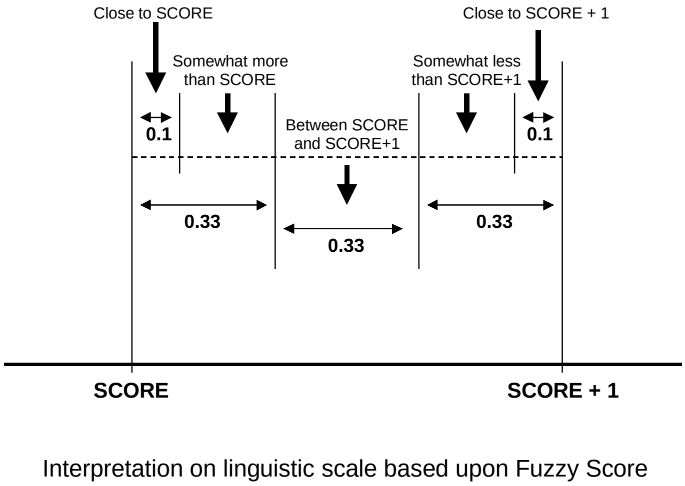

Figure 3. Fuzzy triangular number interpretation.

In the diagram (Figure 3) the interval between SCORE and SCORE + 1 is divided into a total of 5 regions each of which is associated with an appropriate linguistic modifier. Thus, as the calculated score moved from SCORE to SCORE + 1 it travels through successive regions in which a particular linguistic modifier applies. Thus, initially it lies in the region *close to SCORE*, moving next to the region *somewhat more than SCORE*, then to a region *between SCORE and SCORE + 1*, then *somewhat less than SCORE + 1* and finally it lies *close to SCORE + 1*.

The implementation of this scheme means that a direct and automated read out is possible for any triangular fuzzy number in terms of its kernel, minimum and maximum values. In effect a linguistic interpretation of the possibilistic space of the score which is intended to convey a meaningful commentary on the score to a reader.

For the fuzzy triangular score 3.303, 4.277, 5.16 interpreted upon the scale:

1. Never employed
2. Seldom employed
3. Sometimes employed

4. Frequently employed
5. Almost always employed
6. Indispensable to task

For the business task 'informal communications', the interpretation is.

> the score for informal communications is a Kernel value of 4.277 , with a Minimum of 3.303 and a Maximum of 5.16 . The Kernel (most likely) value may be interpreted as somewhat more than frequently employed for informal communications. The Minimum (lowest likely) value may be interpreted as somewhat more than sometimes employed for informal communications. The Maximum (greatest likely) value may be interpreted as somewhat more than almost always employed for informal communications.

This is taken from actual data in (Hassall, 1999, Chapter 4) and has been automatically interpreted by a simple software algorithm.

Formulation 1 - discussion and critique

The formulation developed above may be criticised from a number of perspectives. Firstly and obviously the translation of the ordinal scale linked to numerical values as if it were a representation of a continuum expressible as a real number, albeit a fuzzy one, is to treat the scale implicitly as being of interval nature. This has already been noted above but deserves further discussion.

Some of the original and frequently employed examples of fuzzy numbers and fuzzy set application are based upon a scale which clearly relates to a continuum that may validly be expressed as a meaningful number. The best known one is the discussion of set membership in relation to linguistic concepts such as *young*, *old*, *middle aged* etc. Bademar and Gottwald discuss this example in relation to linguistic variables (Bademar & Gottwald, 1995, Chapter 4). In the example, the judgement that someone is *middle aged* will vary over some range, perhaps from late 30s to early 60s but in any event is clearly linked to an underlying numerically valid continuum. Similarly Lewis relates judgements about body temperature (*normal*, *slight fever*, *moderate fever*, *high fever*) to fuzzy triangular envelopes which are very similar in concept and application to the presently proposed formulation (Lewis, 1997 p. 75). But, again, the temperature continuum underlying the linguistic labels is real enough and continuous enough.

There seem to be two responses to this difficulty. Firstly, it may be possible to employ some statistical techniques to the frequency response tables which enable the ordinal scale to be converted into an interval scale with the characteristic linguistic points re-scaled to represent intervals closer to the internal distances implicit within the data. There are a variety of techniques which have been proposed for doing this based upon correspondence analysis. It is not intended to discuss correspondence analysis here except to note that it is essentially a geometrical way of analysing variables based upon understanding how they are distributed in a multi-dimensional space and how the weightings for particular values of variables are grouped. For example Carroll, Green and Schaffer, 1986 discuss a technique for re-scaling the column intervals of a contingency table using correspondence analysis principles (Carroll, Green & Schaffer, 1986); however, their method was subsequently critiqued by Greenacre who questioned a number of the steps taken (Greenacre, 1989). More recently, Bendixen and Sandler have offered a further technique plus some specific examples based upon real survey data (Bendixen & Sandler, 1995). The motivation for these techniques in re-scaling is primarily to provide data which more accurately reflects an underlying interval scale so that various techniques of multivariate analysis (cluster analysis or discriminant analysis for example) can be reliably applied. However, it is important to determine whether this rationale can necessarily be carried over unquestioned into the sphere of fuzzy variables and linguistic scales.

A possible alternative view of the linguistic scale is that the numerical values associated with each point are definitional and represent a transformation of the linguistic space defined by the linguistic referents. In other words we could assume that some idealised linguistic concordance exists which would accurately reflect what someone means when they judge the use of IT facilities to be *frequently employed* as opposed to *sometimes employed* or *seldom employed*. This would enable the linguistic referents to be assigned to positions within a defined interval scale in some sensible fashion. However, we note that such a concordance is never likely to be found, but one thing that can be said for certain is that, if it is, the concordance assigning 4 as the value for frequently employed, 3 as the value for sometimes employed etc. may be obtained by a process of transformation (or re-scaling) from the true scale.

If we accept the scale established as linking our linguistic referents then the triangular fuzzy numbers derived from the analysis of the questionnaire responses must be acknowledged to be transformations themselves, perhaps a little like measures in some artificial linear space. In this case the characterising and parsing, including the automatic interpretation, is also purely based upon the transformed space or scale. We can imagine that, for example, we might transform the fuzzy triangular numbers back into the *true* scale. The form of the triangular numbers will be changed, perhaps the size of the interval between minimum and kernel and kernel and maximum altering, as well as the values of minimum, kernel and maximum. But, even when this is done, there will be important characteristics of the transformed scale which are retained within the true scale. For example, relative positions of the minimum, kernel and maximum between two scores will remain, indeed must remain if we are not suddenly to conclude that *sometimes employed* in fact represents a higher score than *frequently employed*.

Formulation 2 - best hypothesis determination

Krause and Clark offer a discussion of the interpretation of imprecise or vague data based upon a set theoretic argument whereby evidence is weighted in terms of its contribution to the possibility of a particular hypothesis being true (Krause & Clark, 1993, pp. 127-130). A way to think about this approach is to consider that responses to each of the possible judgements in the ordinal/interval scale:

1. Never employed
2. Seldom employed
3. Sometimes employed
4. Frequently employed
5. Almost always employed
6. Indispensable to task

represents a form of imprecise *sensor* and that the number of responses for each yields a weight distribution across these sensors.

To expand slightly, a score of 3 is a vote from the *3 sensor* in favour of the hypothesis *sometimes employed* but, because this sensor is assumed to be offering only an imprecise datum it might also be expected to offer support to (at least) the adjoining hypotheses as well. In the Krause and Clark formulation the weightings therefore offer a range of supports for each of a number of hypotheses and the most likely hypothesis is the one which has the best support.

In interpreting the frequency table of responses then, it is proposed that the best supported hypothesis can be selected by taking the weighted sums of support for each hypothesis represented by each point on the ordinal/interval scale and adding them. This is conceptually similar to determining the modal value for the distribution but with the assumption that a vote for, e.g., *sometimes employed*, because of the imprecise nature of the data, also carries a level of support for the next lowest (*seldom employed*) and next highest (*frequently employed*) in the scale. So the total support for each hypothesis is the total weighted support included that for the adjacent hypothesis. Thus, from our frequency table of responses we can expect to get a single statement of the hypothesis that is best supported for the particular question being posed.

Conceptually, it is not difficult to come up with a common sense argument in favour of this strategy, which represents a form of approximation to where the centre of gravity of the frequency distribution is. Suppose 10 persons scored (e.g.) point 2 on our scale but 6 persons each scored 4, 5 and 6 respectively. The modal value might suggest that 2 (*seldom employed*) was the most typical response, the mean value is 3.93 (close to *frequently employed*). but, the interpretation proposed will yield 5 (*almost always employed*) as the best hypothesis. This places the best hypothesis amongst the part of the distribution with the highest concentration (weight) of votes.

Discussion of alternative formulations

The two formulations discussed in the preceding sections are both based upon the idea that when respondents score the answer to a survey question on an ordinal or interval scale they are providing an imprecise judgement in relation to a range of hypothetical statements. Implicitly therefore, when a vote is received for a particular statement, this automatically entails a level of support for those hypothesis which, in judgement terms lie immediately adjacent to the one being voted for. This principle is central to the idea of fuzziness in data derived in such a manner and the two formulations have been built upon this framework. However, the two formulations differ in the way in which this principle is exploited.

Firstly, Formulation 1, fuzzy triangular numbers, can be seen as a way of attempting to *retain* information from the frequency table of responses so that something of the shape (or envelope) of responses may be presented at output. Moreover, because it results in a number upon which appropriate arithmetical operations can be performed there is the possibility of various combinations and comparisons of data being produced. In terms of including support for adjacent hypotheses which are "fuzzily" implicit within a particular voted for hypothesis, the fuzzy triangular score has the effect of producing a least optimistic from the *mean minima* for *all* hypotheses, most likely (kernel) from the *mean* for *all* hypotheses and most optimistic from the *mean maxima* for *all* hypotheses Thus, in retaining fuzzy minima and maxima for all hypotheses in the final score, the formulation incorporates the extremes of the fuzzy judgement ranges implied by the responses.

By contrast, Formulation 2, best hypothesis, is a way of *reducing* information available within the frequency table responses. In this case each hypothesis may be supported by votes for immediately adjacent hypotheses but not other hypotheses. This has the effect of producing, in effect, a *forced* best hypothesis which tends to exclude support from extremes of the fuzzy judgement ranges of the responses.

At this stage in the development of the approach it is not possible to say which formulation may offer the most useful output for evaluation purposes, although it seems likely that this may vary with application. However, it does seem likely that the best hypothesis formulation may be useful for applications involving decisions about outcomes from survey research and that fuzzy triangular score may, in contrast, be more useful as an input to qualitative interpretation.

Applications

A variety of applications of both formulations are possible, some of which are briefly discussed below.

Computable measures from limited data

When survey data is available for limited numbers of respondents there can be a difficulty in performing useful analyses. Often, as has been discussed above, such data do not reflect an underlying continuous random variable scale and so only limited non-parametric tests are statistically valid. Moreover, the evaluation of limits of confidence upon measures derived, for example the standard error in a set of observations, cannot be applied for ordinal or interval data. However, if a fuzzy arithmetical approach is taken, all data points contribute (albeit 'fuzzily') to an output figure which may be interpreted against the original, linguistically defined, scale. In this case the interpretation of results must, firstly, be clearly understood to be *qualitatively* different to statistical techniques. However, the techniques describes in this paper, including fuzzy triangular numbers and best hypothesis determination, offer a way of dealing with small sets of data on judgements made by respondents in a consistent manner which acknowledges intrinsic uncertainty in the frame of discernment.

An example of survey data and analysis using the fuzzy triangular score approach is covered in Hassall (1998).

Comparison studies and gap analyses

Comparisons using fuzzy triangular scores derived from questionnaire data offer the possibility of retaining something of the relative distributions of scoring/judgements between different sets of data. For example, whilst the kernel difference between two TFN scores is identical to the difference in the means of the two sets of data, the minimum and maximum differences (in effect) offer a sort of 'worst case' and 'best case' evaluation at the extremes of the fuzzy judgement range.

An example of survey data and analysis to cover the gap between strategy aims and perceptions of achievement is covered in Hassall and Worrall (1997).

Decision making

The use of the best hypothesis formulation seems most promising in decision situations. It is also useful in conjunction with the triangular fuzzy score approach in that it gives information on the 'shape' of the distribution of responses, particularly, whether the responses are grouped or split into regions on the scale. The use of best hypothesis and next best hypothesis, alternatively a complete ranking of hypotheses, offers a useful decision tool. In situations with limited amounts of data, and if the basic formulation of fuzzy scoring upon and interval range is accepted, it is possible to derive a conclusive statement based upon the linguistic referents employed: thus, for example, in Hassall (1999) to conclude that the best statements that can be made about a particular IT systems is that it is 'Never Used' for a particular function.

Qualitative evaluation

A recurring issue at the boundaries of epistemology in information systems and in other management areas, is that of qualitative versos quantitative methods of evaluation. By linking a mathematically rigorous way of treating uncertainty and likelihood to linguistic referents of respondents judgements, the approach described in this paper offers a way of treating questionnaire data that retains both qualitative and quantitative aspects.

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